

An Exceptional Electroweak Model

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Abstract

We consider a gauge extension of the electroweak sector of the Standard Model based on the group $G_2 \times SU(2) \times U(1)$. The exceptional group G_2 is the smallest rank two group that contains $SU(3)$ as a subgroup; the $SU(3)$ prediction $\sin^2 \theta_w = 1/4$ follows approximately in this model if the couplings of the additional $SU(2)$ and $U(1)$ factors are sufficiently large. We study the symmetry breaking sector of the model, the bounds from precision electroweak constraints and the mass spectrum of exotic gauge bosons that may be produced at future colliders. We also discuss an $SU(3)$ electroweak model in which a vector-like sector is included explicitly to facilitate the decays of otherwise stable exotic states. The models considered here represent plausible extensions of the minimal $SU(3)$ electroweak model with potentially distinctive TeV-scale phenomenology.

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I. INTRODUCTION

A major paradigm in the study of physics beyond the Standard Model is the assumption that the forces of nature should have a simple, unified description at high energies. Four-dimensional Grand Unified Theories (GUTs) achieve this goal by embedding the Standard Model gauge group into a larger group, such as $SU(5)$ or $SO(10)$. Gauge invariance requires that the matter and Higgs fields of the Standard Model appear somewhere within complete representations of the unified group. Perhaps the most compelling “experimental” evidence in favor of grand unification is the observation that the Standard Model gauge couplings do indeed unify around 2×10^{16} GeV, at least if the low-energy particle content is that of the Minimal Supersymmetric Standard Model, and an appropriate normalization of hypercharge is assumed. It is not uncommon for practitioners in the field to remark that it would be surprising if this unification of coupling constants turns out to be purely accidental.

In the same spirit, it is intriguing that the $SU(2)$ and $U(1)$ gauge couplings in the minimal Standard Model, g and g' , satisfy relation $g \approx \sqrt{3} g'$, or equivalently

$$\sin^2 \theta_w \approx 1/4 \ , \quad (1.1)$$

around 4 TeV. The possibility that this might not be accidental has motivated the exploration of models in which there is a partial unification of the electroweak gauge groups at the TeV scale [1, 2, 3, 4, 5]. Specifically, Eq. (1.1) is reproduced if $SU(2) \times U(1)$ is embedded in the group $SU(3)$ and the normalization of hypercharge is chosen to be consistent with the branching rule

$$\mathbf{3} = \mathbf{2}_{1/2} + \mathbf{1}_{-1} \ . \quad (1.2)$$

The $SU(2) \times U(1)$ representations on the right-hand-side of Eq. (1.2) allow for the embedding of the left- and right-handed leptons of the Standard Model,

$$\mathbf{3}_R \sim \begin{pmatrix} (e_L)^c \\ -(\nu_L)^c \\ e_R \end{pmatrix} \ , \quad (1.3)$$

where the superscript c represents charge conjugation. The immediate problem with $SU(3)$ electroweak unification (aside from anomaly cancellation) is that there are no $SU(3)$ representations that contain the quark fields of the Standard Model.

One solution to this problem is to extend the gauge group to

$$G = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1), \quad (1.4)$$

with gauge couplings g_3 , \tilde{g} and \tilde{g}' . Let $\text{SU}(2)_0 \times \text{U}(1)_0$ represent the subgroup of $\text{SU}(3)$ defined in our previous example. If the electroweak gauge group of the Standard Model is now identified as the diagonal subgroup of $\text{SU}(2)_0 \times \text{U}(1)_0$ and the additional $\text{SU}(2) \times \text{U}(1)$ factors in Eq. (1.4), then one finds that

$$\begin{aligned} \frac{1}{g^2} &= \frac{1}{g_3^2} + \frac{1}{\tilde{g}^2} \\ \frac{1}{g'^2} &= \frac{3}{g_3^2} + \frac{1}{\tilde{g}'^2}, \end{aligned} \quad (1.5)$$

where g and g' are the Standard Model $\text{SU}(2)_W$ and $\text{U}(1)_Y$ gauge couplings. The prediction that $\sin^2 \theta_w \approx 1/4$ follows if \tilde{g} and \tilde{g}' are much larger than g_3 . Standard Model fields transform only under the additional $\text{SU}(2) \times \text{U}(1)$ factors and are given their conventional charge assignments. All Standard Model fields can therefore be incorporated without anomalies. Without embedding the leptons into a $\mathbf{3}$ of $\text{SU}(3)$, the identification of hypercharge is fixed by the charge assignment¹ of the Higgs field Σ whose vacuum expectation value (vev) breaks $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \rightarrow \text{SU}(2)_W \times \text{U}(1)_Y$:

$$\Sigma \sim (\mathbf{3}, \bar{\mathbf{2}}_{-1/2}) \quad , \quad \langle \Sigma \rangle = \begin{pmatrix} M & 0 \\ 0 & M \\ 0 & 0 \end{pmatrix}. \quad (1.6)$$

This $\text{SU}(3)$ electroweak model, proposed originally by Kaplan and Dimopolous [2], represents a simple gauge extension of the Standard Model that provides an interesting physical prediction, Eq. (1.1), over a wide range of the model's parameter space. It is interesting to note that the gauge structure in Eq. (1.4) follows from the two-site deconstruction of a five-dimensional (5D) $\text{SU}(3)$ unified gauge theory with symmetry breaking imposed via boundary conditions. The 4D $\text{SU}(3)$ electroweak model was studied in phenomenological detail by Csáki *et al.* [3], and motivated the study of 5D $\text{SU}(3)$ electroweak unification by a number of authors [4, 5].

¹ Note that $\bar{\mathbf{2}}$ is isomorphic to $\mathbf{2}$. See Eq. (2.44).

It is natural to consider other simple 4D models with the structure $G_U \times \text{SU}(2) \times \text{U}(1)$ that predict $\sin^2 \theta_w \approx 1/4$. Of course, a theory can be constructed for any G_U that contains $\text{SU}(3)$ as a subgroup; the larger the group G_U , the more exotic states one expects at the scale of symmetry breaking. Here we focus on the next-to-minimal choice for the group G_U . The only groups of rank 2 that contain an $\text{SU}(3)$ subgroup are the groups G_2 and $\text{SU}(3)$ itself. Moreover, the exceptional group G_2 has 14 generators, a number smaller than that of any Lie group of rank greater than 2 that contains an $\text{SU}(3)$ subgroup. Motivated by these observations, we present a $\text{G}_2 \times \text{SU}(2) \times \text{U}(1)$ model that predicts $\sin^2 \theta_w \approx 1/4$, and we study the constraints on the parameter space of the model from precision electroweak observables. The group G_2 has been used before in extensions of the electroweak sector of the Standard Model [6, 7], most notably in the six-dimensional gauge-Higgs unification model of Ref. [7]. By contrast, the model studied here corresponds to the deconstruction of a 5D unified G_2 model with gauge symmetry broken by (non-orbifold) boundary conditions. Nonetheless, the analysis here will proceed from a purely four-dimensional perspective.

One issue that must be considered in models with the gauge group structure $G_U \times \text{SU}(2) \times \text{U}(1)$ is the appearance of exotic, stable charged states [3]. For example, the charged gauge bosons in G_U that are not contained in the $\text{SU}(2)_0 \times \text{U}(1)_0$ subgroup potentially fall in this category. New stable charged particles are severely constrained by heavy isotope searches [8] as well as cosmological bounds [9]. Enlarging G_U from $\text{SU}(3)$ to G_2 (or to any larger gauge group) only exacerbates the problem. We first address this issue in the context of an $\text{SU}(3)$ model; we introduce vector-like leptons into which exotic bosons may decay, and include new Higgs fields that allow these vector-like states to mix with their Standard Model counterparts. The inclusion of additional fields and vevs alters that analysis of unification and electroweak constraints described in Ref. [3]. We determine the constraints on this alternative $\text{SU}(3)$ electroweak model, taking into account updated electroweak data from LEP II that was not available at the time of Ref. [3]. With this machinery in place, we find the analogous constraints on the $\text{G}_2 \times \text{SU}(2) \times \text{U}(1)$ model. In the $\text{SU}(3)$ model, we will explicitly integrate out the vector-like sector to induce a set of higher-dimension operators that allow otherwise stable states to decay. In the G_2 model, we will construct a similar set of operators directly. Interestingly, both the $\text{SU}(3)$ and G_2 -based models have exotic, doubly-charged gauge bosons that may be long-lived; in principle, these states could

travel a macroscopic distance before undergoing a lepton-flavor-violating decay. This would be a remarkable experimental signature. Unfortunately, one cannot determine the lifetime without specifying parameters that are not determined in the low-energy theory.

Our paper is organized as follows. In Section II, we define the $SU(3) \times SU(2) \times U(1)$ and the $G_2 \times SU(2) \times U(1)$ models that we study in this paper. In Section III, we find the parameter space of these models that is allowed by precision electroweak constraints. In Section IV we discuss the effective interactions that contribute to the decays of heavy gauge boson states. In Section V, we summarize our conclusions.

II. THE MODELS

A. $G_U = SU(3)$

In this section, we extend the matter and Higgs field content of the minimal $SU(3)$ electroweak model [2, 3]. Our symmetry-breaking sector consists of two fields, Σ and χ :

$$\Sigma \sim (\mathbf{3}, \bar{\mathbf{2}}_{-1/2}), \quad \langle \Sigma \rangle = \begin{pmatrix} M & 0 \\ 0 & M \\ 0 & 0 \end{pmatrix}, \quad (2.1)$$

$$\chi \sim (\mathbf{3}, \mathbf{1}_1), \quad \langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ x M \end{pmatrix}. \quad (2.2)$$

The parameter M sets the scale of symmetry breaking, while x indicates the ratio of the χ and Σ vevs. The pattern of vevs shown breaks the gauge group down to the diagonal subgroup of $SU(2)_0 \times U(1)_0$ and $SU(2) \times U(1)$. We identify the unbroken symmetry as the electroweak gauge group of the Standard Model, $SU(2)_W \times U(1)_Y$. In addition, we include n_F pairs of vector-like fermions

$$\psi_L^a \sim \psi_R^a \sim \mathbf{3}, \quad a = 1 \dots n_F, \quad (2.3)$$

with mass M_F . The mass scale M_F is a free parameter that we will assume is either of the same order as M or much heavier. Any of the $SU(3)$ gauge bosons can decay to pairs

of the vector-like fermions, if the decay is kinematically allowed. More importantly, each component of ψ will mix with a Standard Model lepton field as a consequence of the choice of Higgs field representations in Eqs. (2.1) and (2.2). The portion of the Lagrangian involving the vector-like fields is given by

$$\mathcal{L} = \bar{\psi}(i \not{D} - M_F)\psi - [\bar{\psi}_L \Sigma \lambda^\ell \ell_R + \bar{\psi}_L \chi \lambda^e e_R + \text{h.c.}] \quad , \quad (2.4)$$

where ℓ_R and e_R refer to Standard Model $SU(2)_W$ doublet and singlet leptons, appropriately charge conjugated as in Eq. (1.3); λ^ℓ and λ^e are Yukawa matrices that mix the heavy and light states. Eq (2.4) assures that the exotic gauge fields will decay ultimately to Standard Model leptons. The simplest way to see this is to integrate out the vector-like matter and study the resulting effective interactions. This will be done in Section IV. For now, we finish defining the model, treating n_F and M_F as free parameters.

The $SU(3)$ gauge bosons $A = A^a T^a$ transform in the adjoint representation, which decomposes under $SU(2) \times U(1)$ as

$$\mathbf{8} = \mathbf{3}_0 + \mathbf{1}_0 + \mathbf{2}_{3/2} + \mathbf{2}_{-3/2}. \quad (2.5)$$

The $\mathbf{2}_{3/2}$ and $\mathbf{2}_{-3/2}$ representations appear in the Lagrangian as a single complex doublet. Taking into account the additional $SU(2) \times U(1)$ factors, we deduce that the gauge bosons in the model fall in the following $SU(2)_W \times U(1)_Y$ representations:

$$\mathbf{3}_0 + \mathbf{1}_0 + \mathbf{3}_0 + \mathbf{1}_0 + \mathbf{2}_{\pm 3/2} \quad . \quad (2.6)$$

The mass eigenstate $\mathbf{3}_0$ fields arise from mixing of the $SU(2)$ gauge fields \widetilde{W}^a and the $SU(2)_0$ gauge fields A^a , for $a = 1, 2, 3$. In the (A^a, \widetilde{W}^a) basis, the mass squared matrix is

$$M^2 \begin{pmatrix} g_3^2 & -g_3 \tilde{g} \\ -g_3 \tilde{g} & \tilde{g}^2 \end{pmatrix}. \quad (2.7)$$

Therefore, one obtains the mass eigenstates

$$W_L^a = c_\phi A^a - s_\phi \widetilde{W}^a \quad (2.8)$$

$$W_H^a = s_\phi A^a + c_\phi \widetilde{W}^a \quad (2.9)$$

with

$$s_\phi = \frac{-g_3}{\sqrt{g_3^2 + \tilde{g}^2}} \quad \text{and} \quad c_\phi = \frac{\tilde{g}}{\sqrt{g_3^2 + \tilde{g}^2}} \quad , \quad (2.10)$$

and the masses

$$M_{W_L} = 0 \quad (2.11)$$

$$M_{W_H} = (g_3^2 + \tilde{g}^2)^{1/2} M. \quad (2.12)$$

Note that our notation follows that of Ref. [3]. The mass eigenstate $\mathbf{1}_0$ fields arise from the mixing of the U(1) field \tilde{B} and the U(1)₀ field A^8 . In the (A^8, \tilde{B}) basis, the mass squared matrix is

$$(1 + 2x^2)M^2 \begin{pmatrix} g_3^2/3 & -g_3\tilde{g}'/\sqrt{3} \\ -g_3\tilde{g}'/\sqrt{3} & \tilde{g}'^2 \end{pmatrix}. \quad (2.13)$$

One immediately obtains the mass eigenstates

$$B_L = c_\psi A^8 - s_\psi \tilde{B} \quad (2.14)$$

$$B_H = s_\psi A^8 + c_\psi \tilde{B} , \quad (2.15)$$

with

$$s_\psi = \frac{-g_3}{\sqrt{g_3^2 + 3\tilde{g}'^2}}, \quad \text{and} \quad c_\psi = \frac{\sqrt{3}\tilde{g}'}{\sqrt{g_3^2 + 3\tilde{g}'^2}} , \quad (2.16)$$

and the masses

$$M_{B_L} = 0 \quad (2.17)$$

$$M_{B_H} = (1 + 2x^2)^{1/2} \left(\frac{g_3^2}{3} + \tilde{g}'^2 \right)^{1/2} M. \quad (2.18)$$

Finally, the $\mathbf{2}_{\pm 3/2}$ state is formed from the remaining components of the SU(3) adjoint, A^a for $a = 4, 5, 6, 7$. Its mass is given by

$$M_{3/2} = \frac{1}{\sqrt{2}}(1 + x^2)^{1/2} g_3 M, \quad (2.19)$$

where the subscript indicates the hypercharge of the state. For $g_3 \ll \tilde{g}, \tilde{g}'$, the $\mathbf{2}_{\pm 3/2}$ gauge bosons will be significantly lighter than the other massive bosons, W_H^a and B_H .

The remainder of the exotic particle spectrum originates from the Higgs fields Σ and χ . Given the branching rule $\mathbf{3} = \mathbf{2}_{1/2} + \mathbf{1}_{-1}$ under the SU(2)₀ × U(1)₀ subgroup, one deduces the following SU(2)_W × U(1)_Y decompositions of the scalar fields:

$$\Sigma \rightarrow (\mathbf{2}_{1/2} + \mathbf{1}_{-1}) \otimes \mathbf{2}_{-1/2} = \mathbf{3}_0 + \mathbf{1}_0 + \mathbf{2}_{-3/2} , \quad (2.20)$$

$$\chi \rightarrow (\mathbf{2}_{1/2} + \mathbf{1}_{-1}) \otimes \mathbf{1}_1 = \mathbf{2}_{3/2} + \mathbf{1}_0. \quad (2.21)$$

Therefore, the symmetry-breaking sector consists of complex scalar fields in the following $SU(2)_W \times U(1)_Y$ representations

$$\mathbf{3}_0 + \mathbf{1}_0 + \mathbf{1}_0 + \mathbf{2}_{3/2} + \mathbf{2}_{-3/2}. \quad (2.22)$$

The spectrum of the scalar sector is model dependent. Generically, one expects that all physical scalar states should obtain masses of order the symmetry-breaking scale M . Here we will verify this statement by finding a local minimum of the scalar potential.

To construct a potential, we first list the possible gauge-invariant operators involving Σ and χ , up to quartic order in these fields. We find

$$t_1 = m^2 \text{Tr} \Sigma^\dagger \Sigma, \quad (2.23)$$

$$t_2 = \text{Tr} \Sigma^\dagger \Sigma \text{Tr} \Sigma^\dagger \Sigma, \quad (2.24)$$

$$t_3 = \text{Tr} \Sigma^\dagger \Sigma \Sigma^\dagger \Sigma, \quad (2.25)$$

$$t_4 = \text{Tr} \Sigma \epsilon \Sigma^T \Sigma^* \epsilon \Sigma^\dagger, \quad (2.26)$$

$$t_5 = m^2 \chi^\dagger \chi, \quad (2.27)$$

$$t_6 = \chi^\dagger \chi \chi^\dagger \chi, \quad (2.28)$$

$$t_7 = \chi^\dagger \Sigma \Sigma^\dagger \chi, \quad (2.29)$$

$$t_8 = m \Sigma_\alpha^i \Sigma_\beta^j \epsilon^{\alpha\beta} \chi^k \epsilon_{ijk} + \text{h.c.}, \quad (2.30)$$

$$t_9 = \chi^\dagger \chi \text{Tr} \Sigma^\dagger \Sigma, \quad (2.31)$$

where

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad (2.32)$$

and where m is a mass of the same order as the desired symmetry-breaking scale. The potential is an arbitrary linear combination of these invariant terms, with coefficients α_i ,

$$V = \sum_{i=1}^9 \alpha_i t_i. \quad (2.33)$$

TABLE I: Spectrum of physical scalars in the SU(3) model, in units of m , for the example parameter choice described in the text.

state	mass
$\mathbf{3}_0$	2.06
$\mathbf{2}_{3/2}$	1.30
$\mathbf{1}_0$	1.55
$\mathbf{1}_0$	1.40
$\mathbf{1}_0$	1.32

To find a local minimum, we perform a constrained minimization. Substituting Eqs (2.1) and (2.2) into the potential, we minimize the resulting function V_0 , given by

$$V_0 = 2\alpha_1 M^2 + (4\alpha_2 + 2\alpha_3 - 2\alpha_4)M^4 + \alpha_5 M^2 x^2 + \alpha_6 M^4 x^4 + 4\alpha_8 M^3 x + 2\alpha_9 M^4 x^2. \quad (2.34)$$

For an example, setting $(\alpha_1, \dots, \alpha_9) = (-1, 1.1, 1.2, 1.4, -1.3, 0.9, 0.7, -0.8, 0.5)$, we find that the global minimum of the potential over the parameters M and x is at $(M, x) = (0.71994m, 1.32919)$. We confirm that this point is a minimum by studying the scalar mass squared matrix

$$M_{ij}^2 = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j}, \quad (2.35)$$

where the ϕ_i denote the real scalar degrees of freedom in the fields Σ and χ , with $1 \leq i \leq 18$. The squared masses are all positive, as shown in Table I, and have the correct multiplicity to occupy complete representations of the unbroken gauge group. For this choice of parameters α_i , we also confirm that there are eight zero eigenvalues, corresponding precisely to the $12-4$ broken generators in the spontaneous breaking $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \rightarrow \text{SU}(2)_W \times \text{U}(1)_Y$.

B. $G_U = \text{G}_2$

We construct the model with $G_U = \text{G}_2$ in analogy to the SU(3) model of Section II A, though in this case we do not include vector-like matter. The group G_2 contains SU(3) as a maximal subgroup; the fundamental representation of G_2 is seven dimensional and

decomposes under this SU(3) as

$$\mathbf{7} = \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1} . \quad (2.36)$$

This result suggests the natural generalization of the Higgs sector of the SU(3) theory: Symmetry breaking is achieved via two fields, Σ and χ with the following quantum numbers and vevs:

$$\Sigma \sim (\mathbf{7}, \bar{\mathbf{2}}_{-1/2}) \quad , \quad \langle \Sigma \rangle = \begin{pmatrix} M & 0 \\ 0 & M \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad , \quad \chi \sim (\mathbf{7}, \mathbf{1}_1) \quad , \quad \langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ x M \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} . \quad (2.37)$$

With the vevs shown in Eq. (2.37), G_2 is broken to $SU(2)_W \times U(1)_Y$. The adjoint representation of G_2 is 14-dimensional and decomposes as $\mathbf{8} + \mathbf{3} + \bar{\mathbf{3}}$ under the SU(3) subgroup. This immediately tells us that the electroweak quantum numbers of the gauge bosons are given by,

$$\mathbf{14} = (\mathbf{3}_0 + \mathbf{1}_0 + \mathbf{2}_{3/2} + \mathbf{2}_{-3/2}) + (\mathbf{2}_{1/2} + \mathbf{1}_{-1}) + (\mathbf{2}_{-1/2} + \mathbf{1}_1) . \quad (2.38)$$

Again, pairs of representations with opposite hypercharges correspond to complex vector fields. With M and x defined as in Eq. (2.37), the mass spectrum of W_H , W_L , B_H , B_L , and the exotic $\mathbf{2}_{\pm 3/2}$ states, as well as the mixing angles ϕ and ψ , are precisely the same as in the SU(3) model of Section II A, with the identification

$$g_3 = g_2 / \sqrt{2} \quad , \quad (2.39)$$

where g_2 is the G_2 gauge coupling. We find that the masses of the new SU(2) doublet and singlet bosons are given by

$$M_{1/2} = \frac{1}{\sqrt{6}} (3 + x^2)^{1/2} g_3 M \quad , \quad (2.40)$$

$$M_1 = \frac{1}{\sqrt{3}} (1 + x^2)^{1/2} g_3 M \quad , \quad (2.41)$$

where the subscripts again refer to the hypercharges of the states. Note that the $\mathbf{1}_{\pm 1}$ states are always lighter than the $\mathbf{2}_{\pm 3/2}$ bosons of the SU(3) model. We postpone a discussion of the sector that facilitates the decays of these states until Section IV.

We now consider the scalar sector of the model. It follows from the decompositions in Eqs. (1.2) and (2.36) that the Σ and χ fields contain the following $SU(2)_W \times U(1)_Y$ reps:

$$\Sigma = (\mathbf{3}_0 + \mathbf{1}_0 + \mathbf{2}_{-3/2}) + (\mathbf{3}_{-1} + \mathbf{1}_{-1} + \mathbf{2}_{1/2}) + \mathbf{2}_{-1/2} , \quad (2.42)$$

$$\chi = (\mathbf{2}_{3/2} + \mathbf{1}_0) + (\mathbf{2}_{1/2} + \mathbf{1}_2) + \mathbf{1}_1 . \quad (2.43)$$

We will show that there are local minima of a G_2 invariant potential with the desired pattern of symmetry-breaking vevs. Compared to the SU(3) model, however, the construction of invariant interactions is more interesting. In constructing the potential for the SU(3) model, we used the fact that SU(2) has only real representations, so that

$$\mathbf{2} \sim \epsilon \mathbf{2}^* . \quad (2.44)$$

where $\epsilon = i\sigma^2$. The group G_2 also has only real representations. One finds that

$$\mathbf{7} \sim S \mathbf{7}^* , \quad (2.45)$$

where S is the matrix

$$S = \begin{pmatrix} 0 & \mathbb{1} & 0 \\ \mathbb{1} & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} , \quad (2.46)$$

and where $\mathbb{1}$ is a three-by-three identity matrix. Eq. (2.45) allows the construction of many more invariants compared to the potential in the SU(3) model. Finally, the cubic invariant in the potential for the SU(3) model, which exists because it is possible to make a singlet out of three triplets, has a natural generalization in the G_2 model. We note that the tensor product [11]

$$\mathbf{7} \times \mathbf{7} = \mathbf{1} + \mathbf{7} + \mathbf{14} + \mathbf{27} \quad (2.47)$$

implies that $\mathbf{7}^3$ also contains a singlet. In SU(3), the appropriate Clebsch-Gordan coefficients for making a singlet from three triplets is the three-index epsilon tensor ϵ_{ijk} ; in G_2 , we find that the analogous object is a totally antisymmetric tensor C_{ijk} , with

$$C_{714} = C_{725} = C_{736} = 1 , \quad C_{123} = C_{456} = -\sqrt{2} . \quad (2.48)$$

All components that are not related to these by total antisymmetry are vanishing. The nonvanishing components of C_{ijk} can be understood by considering the transformation properties of a cubic invariant, for example

$$\psi_1^i \psi_2^j \psi_3^k C_{ijk} \ , \quad (2.49)$$

where $\psi_i \sim \mathbf{7}$, under the $SU(3)$ subgroup. Referring to the basis defined in Appendix A, the components with one index equal to 7 couple a $\mathbf{3}$ in one $\mathbf{7}$ to a $\bar{\mathbf{3}}$ in another and combine this with a singlet from the third. The components with indices 123 (456) provide an $SU(3)$ epsilon tensor that couples three $\mathbf{3}$'s ($\bar{\mathbf{3}}$'s), with one originating from each of the $\mathbf{7}$'s. All the other elements of C_{ijk} that are not related to these by antisymmetry must be zero since they would not lead to a result that is invariant under the $SU(3)$ subgroup. The relative normalization between the components that lead to different $SU(3)$ invariants (*i.e.*, the factor of $-\sqrt{2}$ in Eq. (2.48)) is fixed by the condition that Eq. (2.49) remain invariant under the action of the full group. We have checked this for infinitesimal transformations; our result for C_{ijk} satisfies

$$C_{i'jk}(T^a)_i^{i'} + C_{ij'k}(T^a)_j^{j'} + C_{ijk'}(T^a)_k^{k'} = 0 \quad (2.50)$$

for $a = 1, \dots, 14$, where the T^a are the G_2 generators given in the Appendix A.

In analogy to Eq. (2.33), we may write the Σ - χ potential as

$$V = \sum_{i=1}^{12} \beta_i u_i \ , \quad (2.51)$$

where the β_i are parameters and the u_i are the following $G_2 \times SU(2) \times U(1)$ invariants:

$$u_1 = m^2 \text{Tr} \Sigma^\dagger \Sigma \quad (2.52)$$

$$u_2 = \text{Tr} \Sigma^\dagger \Sigma \text{Tr} \Sigma^\dagger \Sigma \quad (2.53)$$

$$u_3 = \text{Tr} \Sigma^\dagger \Sigma \Sigma^\dagger \Sigma \quad (2.54)$$

$$u_4 = \text{Tr} \Sigma^T S \Sigma \Sigma^\dagger S \Sigma^* \quad (2.55)$$

$$u_5 = \text{Tr} \Sigma \epsilon \Sigma^T \Sigma^* \epsilon \Sigma^\dagger \quad (2.56)$$

$$u_6 = \text{Tr} \Sigma^T S \Sigma \epsilon \text{Tr} \Sigma^\dagger S \Sigma^* \epsilon \quad (2.57)$$

$$u_7 = m^2 \chi^\dagger \chi \quad (2.58)$$

TABLE II: Spectrum of physical scalars, in units of m , for the example parameter choice described in the text.

state	mass	state	mass
3₋₁	4.52	1₂	5.24
3₀	6.52	1₁	2.84
2_{3/2}	4.25	1₀	4.03
2_{1/2}	3.03	1₀	3.61
2_{1/2}	2.19	1₀	2.25

$$u_8 = \chi^\dagger \chi \chi^\dagger \chi \quad (2.59)$$

$$u_9 = \chi^T S \chi \chi^\dagger S \chi^* \quad (2.60)$$

$$u_{10} = m \Sigma_\alpha^i \Sigma_\beta^j \epsilon^{\alpha\beta} \chi^k C_{ijk} + \text{h.c.} \quad (2.61)$$

$$u_{11} = \chi^\dagger \Sigma \Sigma^\dagger \chi \quad (2.62)$$

$$u_{12} = \chi^T S \Sigma \Sigma^\dagger S \chi^* \quad (2.63)$$

Eq. (2.51) is a function of 42 real scalar degrees of freedom, and depends on 12 free parameters. A complete study of this potential is beyond the scope of this paper. Nevertheless, we can again show that there are extrema of this potential with the desired symmetry-breaking vevs that correspond to local minima. For this purpose, we first extremize the potential with the constraint that the non-vanishing vevs are those displayed in Eq. (2.37); for these solutions we then determine whether the second-derivative matrix of the potential is positive semi-definite, the same procedure that we employed in the SU(3) model. As an example, for the choice of parameters $(\beta_1 \dots \beta_{12}) = (-1, 0.3, 1.1, 1.0, 1.0, 1.0, -1.0, 0.3, 1.0, 1.0, 1.0, 0.1)$, one obtains $M = 2.79 m$, $x = 1.25$ and the mass spectrum shown in Table II. This parameter choice was random; generically, we don't find any fine-tuning is necessary to find solutions. Since we have established that there is no difficulty in finding appropriate symmetry-breaking vacua in both the SU(3) and G₂ models, we take M and x as free parameters in the phenomenological analysis that follows.

III. ELECTROWEAK CONSTRAINTS

The most important electroweak constraints on models with the symmetry-breaking $G_U \times \text{SU}(2) \times \text{U}(1) \rightarrow \text{SU}(2)_W \times \text{U}(1)_Y$ comes from the tree-level mixing between the $\text{SU}(2)_0 \times \text{U}(1)_0$ and the $\text{SU}(2) \times \text{U}(1)$ gauge bosons, where $\text{SU}(2)_0 \times \text{U}(1)_0 \subset G_U$. The gauge bosons corresponding to the broken generators of the original gauge group do not couple directly to Standard Model matter fields, aside from higher-dimension operators that are generically suppressed. For this reason, the analysis of electroweak constraints on the $\text{SU}(3)$ and G_2 models of interest to us here proceeds in analogy to that of the minimal weak $\text{SU}(3)$ model discussed in Ref. [3]. The present analysis differs from that of the minimal $\text{SU}(3)$ model in three ways: *(i.)* The low-energy effective Lagrangian that one obtains after integrating out the heavy W_H and B_H fields depends on their masses, which differ in our model due to the χ field vev. Our results therefore depend on the additional parameter x defined in Eqs. (2.2) and (2.37) *(ii.)* The presence of the χ field, as well as vector-like matter in the $\text{SU}(3)$ model, alters the beta functions that determine the matching between the low-energy gauge couplings, g and g' , and the couplings of the high-energy theory, g_U , \tilde{g} and \tilde{g}' , via Eq. (1.5). *(iii.)* We use updated data for the electroweak observables that was not available at the time of Ref. [3].

Since our approach follows that of Ref. [3], we will confine our discussion to the way in which the analysis is modified. Letting Φ represent the Higgs field of the minimal Standard Model, corrections to the ordinary W and Z -boson masses follow from study of the kinetic term $(D_\mu \Phi)^\dagger (D^\mu \Phi)$, which includes couplings to both the W_H , W_L , B_H and B_L fields. The heavy fields can be eliminated by applying their equations of motion so that all the quadratic terms can be expressed in terms of the light fields only. Following this approach, we find

$$m_W^2 = \frac{g^2 v^2}{4} \left(1 - \frac{v^2 c_\phi^4}{4M^2} \right) , \quad (3.1)$$

$$m_Z^2 = \frac{v^2}{4} (g^2 + g'^2) \left[1 - \frac{v^2}{4M^2} \left(c_\phi^4 + \frac{1}{1 + 2x^2} c_\psi^4 \right) \right] , \quad (3.2)$$

where $\langle \Phi \rangle = v/\sqrt{2}$ with $v = 246$ GeV. Similarly, exchanges of the W_H and B_H fields alter the couplings of the Standard Model gauge fields to ordinary matter. Since Standard Model fields couple only to the extra $\text{SU}(2) \times \text{U}(1)$ factors, the gauge-matter couplings are

summarized by

$$\mathcal{L}_J = \tilde{g} \tilde{W}_\mu^a J^{a\mu} + \tilde{g}' \tilde{B}_\mu J_Y^\mu , \quad (3.3)$$

where J^a and J_Y are the conventional $\text{SU}(2)_W$ and $\text{U}(1)_Y$ currents of the Standard Model. The fields \tilde{W} and \tilde{B} may be expressed in terms of W_H , W_L , B_H and B_L , and the heavy fields can again be eliminated by use of their equations of motion. We find

$$\begin{aligned} \mathcal{L} = & g \left(1 - \frac{v^2}{4M^2} c_\phi^4 \right) W_L^a \cdot J^a + g' \left(1 - \frac{v^2}{4M^2} \frac{c_\psi^4}{1+2x^2} \right) B_L \cdot J_Y + g' \left(\frac{v^2 c_\phi^4}{4M^2} \right) B_L \cdot J^3 \\ & + g \left(\frac{v^2}{4M^2} \frac{c_\psi^4}{1+2x^2} \right) W_L^3 \cdot J_Y - \left(\frac{c_\phi^4}{2M^2} \right) J^a \cdot J^a - \left(\frac{1}{2M^2} \frac{c_\psi^4}{1+2x^2} \right) J_Y \cdot J_Y . \end{aligned} \quad (3.4)$$

The leading electroweak corrections in our models follow from Eqs. (3.1), (3.2) and (3.4). These results agree with the corresponding expressions in Ref. [3] when $x = 0$, and indicate that the theoretical prediction for any electroweak observable can be obtained from the $x = 0$ result via the substitution

$$c_\psi^4 \longrightarrow \frac{c_\psi^4}{1+2x^2} . \quad (3.5)$$

Theoretical predictions for 22 electroweak observables are given in an Appendix of Ref. [3], and are expressed as a function of two parameters, c_1 and c_2 . These predictions remain valid in our models if we identify

$$c_1 = \frac{c_\phi^4 v^2}{4M^2} \quad \text{and} \quad c_2 = \frac{1}{1+2x^2} \frac{c_\psi^4 v^2}{4M^2} . \quad (3.6)$$

Following the approach of Ref. [3], we construct a chi-squared function for the shifts in electroweak precision observables from their Standard Model values as a function of the parameters c_1 and c_2 . The Standard Model predictions and experimental data are taken from a fit by Langacker and Erler that appears in the 2006 Review of Particle Properties [10]. For convenience, we quote these values in Table III. We compute confidence contours by taking the new theory to be the null hypothesis. Then $\chi^2 - \chi_{min}^2$ is also χ^2 distributed, with two degrees of freedom (the number of parameters, c_1 and c_2). The main difference in the electroweak data that we use in for our fit compared to Ref. [3] is that more recent LEP II results have shifted the central value of the W mass downward. Since the nonstandard contribution to M_W in our models is positive, the parameter space is now more tightly constrained. We illustrate this in Fig. 1, which displays the 68%, 95% and 99% confidence contours from our global fit compared to those in Ref. [3]. The shift in these contours does

TABLE III: Input parameters for the electroweak fit described in the text. The SM column shows central values from a Standard Model fit by Langacker and Erler, appearing in the 2006 Review of Particle Physics [10], in which $m_Z = 91.1874 \pm 0.0021$ GeV, $m_H = 89^{+38}_{-28}$ GeV, $m_t = 172.7 \pm 2.8$ GeV and $\alpha_s(m_Z) = 0.1216 \pm 0.0017$.

Quantity	Experiment	SM	Quantity	Experiment	SM
Γ_Z	2.4952 ± 0.0023	2.4968	$A_e(P_\tau)$	0.1498 ± 0.0049	0.1471
R_e	20.8040 ± 0.0500	20.7560	A_{FB}^b	0.0992 ± 0.0016	0.1031
R_μ	20.7850 ± 0.0330	20.7560	A_{FB}^c	0.0707 ± 0.0035	0.0737
R_τ	20.7640 ± 0.0450	20.8010	A_{LR}	0.15138 ± 0.00216	0.1471
σ_h	41.5410 ± 0.0370	41.4670	M_W	80.403 ± 0.029	80.3760
R_b	0.21629 ± 0.00066	0.21578	M_W/M_Z	0.88173 ± 0.00032	0.8814
R_c	0.1721 ± 0.0030	0.17230	$g_L^2(\nu N \rightarrow \nu X)$	0.30005 ± 0.00137	0.30378
A_{FB}^c	0.0145 ± 0.0025	0.01622	$g_R^2(\nu N \rightarrow \nu X)$	0.03076 ± 0.00110	0.03006
A_{FB}^μ	0.0169 ± 0.0013	0.01622	$g_{eA}(\nu e \rightarrow \nu e)$	-0.5070 ± 0.014	-0.5064
A_{FB}^τ	0.0188 ± 0.0017	0.01622	$g_{eV}(\nu e \rightarrow \nu e)$	-0.040 ± 0.015	-0.0396
$A_\tau(P_\tau)$	0.1439 ± 0.0043	0.1471	$Q_W(Cs)$	-72.62 ± 0.46	-73.17

not lead to a dramatic change in the allowed parameter space of the minimal SU(3) model. The shape of the exclusion region in our models depends more noticeably on the value of the parameter x , as we describe below.

The parameter space of the model may be described in terms of the couplings \tilde{g}' , and \tilde{g} . As in Ref. [3], we define the unification scale M_U as the mass of the heaviest gauge boson, the threshold at which the matching conditions Eq. (1.5) should be applied. The Standard Model gauge couplings $g(M_U)$ and $g'(M_U)$, are determined via the one-loop renormalization group equations

$$\alpha^{-1}(M_U) = \alpha^{-1}(M_Z) + \frac{b_{SM}}{2\pi} \ln \frac{M_U}{M_Z} + \sum_i \frac{b_i}{2\pi} \ln \frac{M_U}{M_i} , \quad (3.7)$$

$$\alpha'^{-1}(M_U) = \alpha'^{-1}(M_Z) + \frac{b'_{SM}}{2\pi} \ln \frac{M_U}{M_Z} + \sum_i \frac{b'_i}{2\pi} \ln \frac{M_U}{M_i} , \quad (3.8)$$

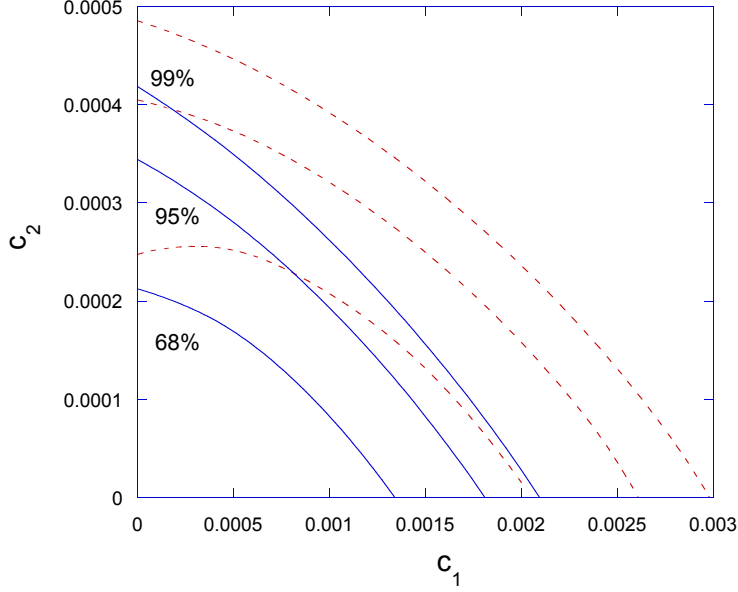


FIG. 1: Confidence level contours in the c_1 - c_2 plane for the electroweak fit described in the text. The dotted lines show the corresponding results for the input data used in Ref. [3].

where M_i is the mass of the i^{th} heavy particle threshold, and b_i the contribution to the beta function. For the heavy gauge bosons, the M_i are proportional to M_U , since the unification scale is identified as the larger of Eq. (2.12) or (2.18); the other heavy boson states are always lighter than this result. The physical scalar components of the Σ and χ Higgs fields are taken to have the same mass as the $\mathbf{2}_{3/2}$ gauge bosons, the same approximation used in Ref. [3]. When vector-like matter is included, the mass scale is separately specified. The values for the beta functions are given in Table IV. If one specifies \tilde{g}' and \tilde{g} , then Eqs. (1.5), (3.7) and (3.8) completely determine M_U and the coupling $g_3(M_U)$. The quantities c_ϕ and c_ψ follow immediately, while the parameter M is known through the identification of M_U with the heaviest gauge boson mass. All the quantities needed to compute the values of c_1 and c_2 are thereby obtained. We implement this procedure numerically to associate each point in the \tilde{g}' - \tilde{g} plane with a point in c_1 - c_2 space; in this way, we determine whether a given point in the model's parameter space is excluded, to any desired confidence level. We show the 95% confidence level exclusion regions in the results that follow.

To complete the analysis, we must specify values of x and, in the SU(3) model, the number of $\mathbf{3} + \overline{\mathbf{3}}$ pairs n_F . Figures 2 and 3 show our results for the matching scale M_U assuming $n_F = 1$ and $M_F = 1$ TeV, allowing x to vary between $1/3$ and 3 . Larger x tends

	SU(3)		G ₂	
states	b_i	b'_i	b_i	b'_i
Standard Model	19/6	-41/6	19/6	-41/6
$\mathbf{1}_{\pm 1}$ vector	-	-	0	7
$\mathbf{2}_{\pm 1/2}$ vector	-	-	7/2	7/2
$\mathbf{2}_{\pm 3/2}$ vector	7/2	63/2	7/2	63/2
physical scalars	-1/2	-3/2	-3/2	-9/2
vector-like	$-2n_F/3$	$-2n_F$	-	-

TABLE IV: Beta functions b_i in Eqs. (3.7) and (3.8). Vector boson beta functions include the contribution from the longitudinal (eaten scalar) component. Physical scalars are assumed to have the same mass as the $\mathbf{2}_{3/2}$ vector bosons. The number of $\mathbf{3} + \bar{\mathbf{3}}$ pairs is given by n_F .

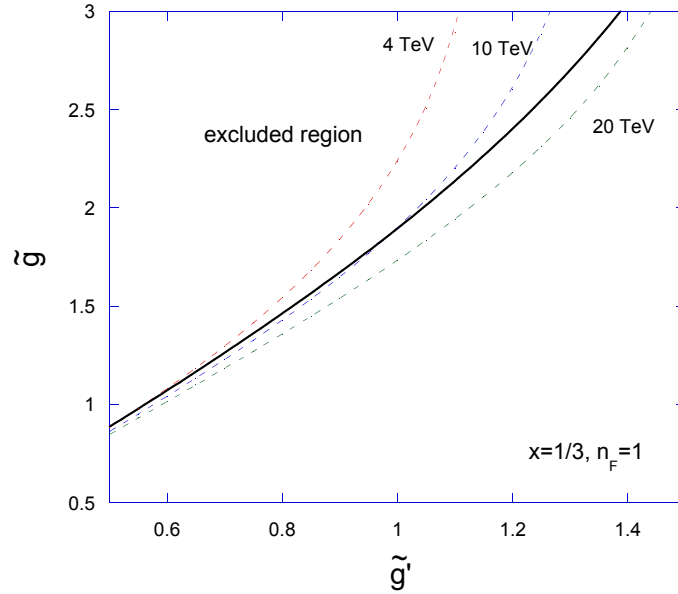


FIG. 2: Contours of the unification scale M_U in the SU(3) model for $x = 1/3$, $n_F = 1$ and $M_F = 1$ TeV. The 95% C.L. exclusion region is also shown.

to exclude smaller values of \tilde{g} ; however, the constant M_U contours and the boundary of the excluded region move in tandem, so that the effect on the smallest allowed value of M_U is relatively mild. It is worth noting that there is an optimal choice $x \approx 1.2$ for which the

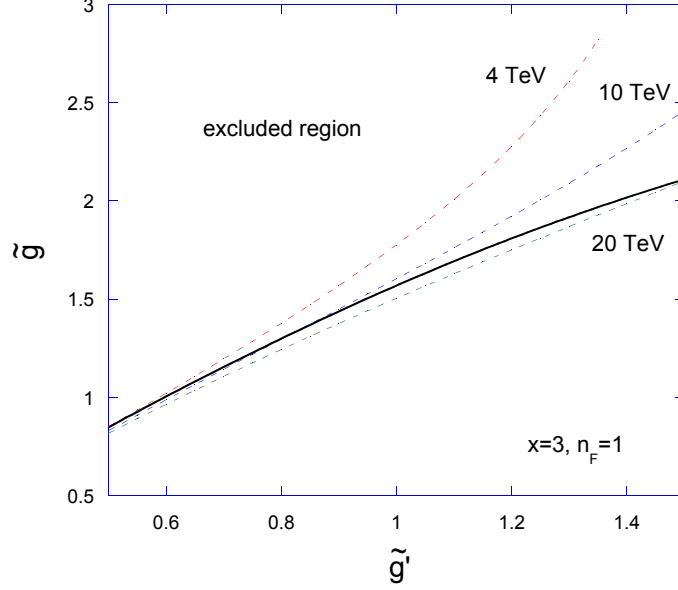


FIG. 3: Contours of the unification scale M_U in the SU(3) model for $x = 3$, $n_F = 1$ and $M_F = 1$ TeV. The 95% C.L. exclusion region is also shown.

$M_U = 10$ TeV contour is within the allowed region for $\tilde{g}' < 1.2$ and $\tilde{g} < 2$, an improvement over the minimal SU(3) model. However, at large values, $x \geq 3$, the exclusion region grows, engulfing the entire $M_U = 10$ TeV contour. Varying the number of heavy fermion pairs between 0 and 3 has a negligible effect on the position of these contours or the excluded region, so we do not provide separate plots. Figure 4 shows mass contours for the $\mathbf{2}_{3/2}$ gauge boson in the SU(3) model, with $n_F = 0$. Generally, we note that $M_{3/2} = 2$ TeV is entirely excluded and $M_{3/2} = 4$ TeV entirely outside the exclusion region. For an optimal value of $x \approx 0.9$, the $M_{3/2} = 3$ TeV curve is completely outside the exclusion region. Again, for $x \geq 3$, the exclusion region becomes large, excluding $M_{3/2} = 4$ TeV as well².

In the case of the G_2 model, we work with $n_F = 0$ since we will arrange for boson decays via higher-dimension operators that do not necessarily arise via the exchange of vector-like matter. The renormalization group analysis differs since there are now additional mass thresholds, as indicated in Table IV. Note that the lightest gauge boson state in the G_2

² In all our figures, $\tilde{\alpha} < 1$ and $\tilde{\alpha}' < 1$ so that our analysis does not extend into the parameter space in which nonperturbative effects (*e.g.*, the formation of fermion condensates) may become important. The ranges in \tilde{g} and \tilde{g}' shown were chosen to coincide with those of Ref. [3], for ease of comparison.

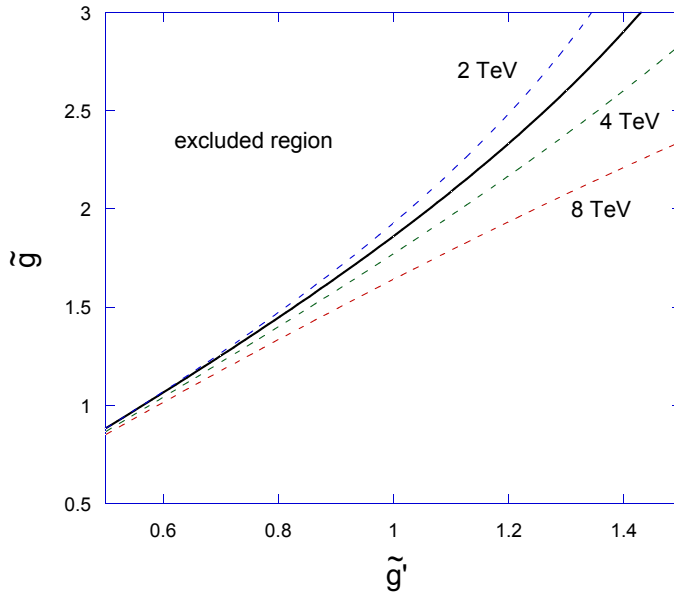


FIG. 4: Contours for $M_{3/2}$, the mass of lightest exotic gauge boson in the SU(3) weak model, assuming $x = 1$ and $n_F = 0$. The 95% C.L. exclusion region is also shown.

model depends on the choice of parameters. With $x \leq 1$, the $\mathbf{1}_{\pm 1}$ field is the lightest; its mass is always smaller by a factor of $\sqrt{2/3}$ relative to the $\mathbf{2}_{\pm 3/2}$ state in the SU(3) model. With $x > 1$, the $\mathbf{2}_{\pm 1/2}$ is the lighter, while for $x = 1$ the $\mathbf{1}_{\pm 1}$ and $\mathbf{2}_{\pm 1/2}$ are degenerate. Figures 5 and 6 show the unification and lightest exotic gauge boson contours, respectively, for the choice $x = 1$. As one might anticipate, the results are qualitatively similar to the SU(3) model, aside from the lighter exotic gauge boson states. The fact that the $m_1 = m_{1/2} = 2$ TeV contour in Fig. 6 is almost contiguous with the exclusion line suggests that these exotic gauge bosons could be pair produced at the LHC. Ranges of minimum allowed gauge boson masses for approximately optimal values of x are shown in Table V. The differing gauge boson spectrum of the G_2 model may therefore make it an easier candidate for direct detection at future collider experiments³.

³ Note that we do not exclude any additional region of the parameter space in our models from the consideration of neutral meson mixing. For example, the W_H contribution to the K_L - K_S mass splitting is suppressed relative to the Standard Model result, by a factor that is well approximated by $2(\tilde{g} \cos \phi / g)^2 (M_W / M_{W_H})^2$, and does not exceed Standard Model hadronic uncertainties anywhere in the region allowed by the precision electroweak constraints.

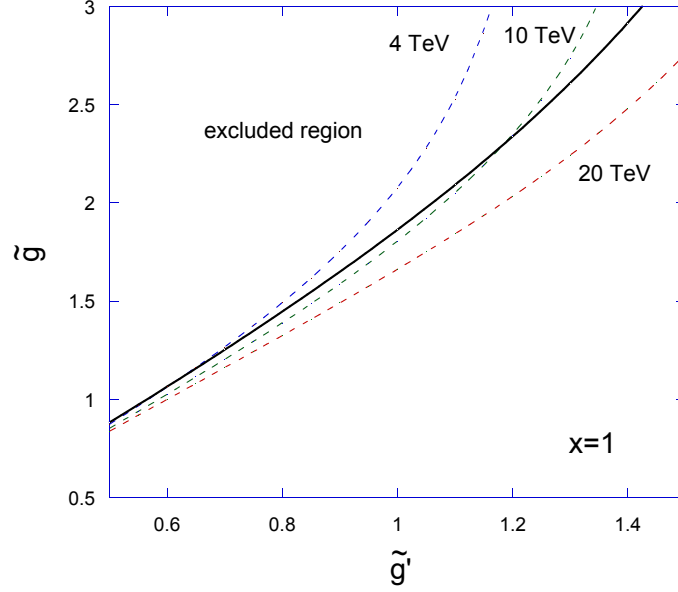


FIG. 5: Contours of the unification scale M_U in the G_2 model for $x = 1$. The 95% C.L. exclusion region is also shown.

TABLE V: Ranges of minimum allowed gauge boson masses in the G_2 model as \tilde{g}' varies from 0.5 to 1.5, for a choice of x near its optimal value. The result in the $SU(3)$ model for the $\mathbf{2}_{\pm 3/2}$ state is the same as in the G_2 model, to the accuracy shown.

State	x	Mass range (TeV)
$\mathbf{2}_{\pm 3/2}$	0.9	2.27 - 2.75
$\mathbf{1}_{\pm 1}$	0.9	1.85 - 2.22
$\mathbf{2}_{\pm 1/2}$	1.5	1.76 - 2.15

IV. DECAYS

In the minimal $SU(3)$ model, the $\mathbf{2}_{\pm 3/2}$ gauge bosons have no direct coupling to Standard Model particles. To avoid unwanted, charged stable states, the model presented in Section II A included vector-like $\mathbf{3} + \bar{\mathbf{3}}$ pairs. The $\mathbf{3}$ has the same $SU(2)_W \times U(1)_Y$ decomposition as a lepton generation in the Standard Model. Mass mixing between the vector-like and Standard Model leptons will occur if there are Higgs fields with the quantum numbers $(\mathbf{3}, \bar{\mathbf{2}}_{1/2}) + (\mathbf{3}, \mathbf{1}_{-1})$ and with vevs in the components that transform as a $\mathbf{1}_0$ under the

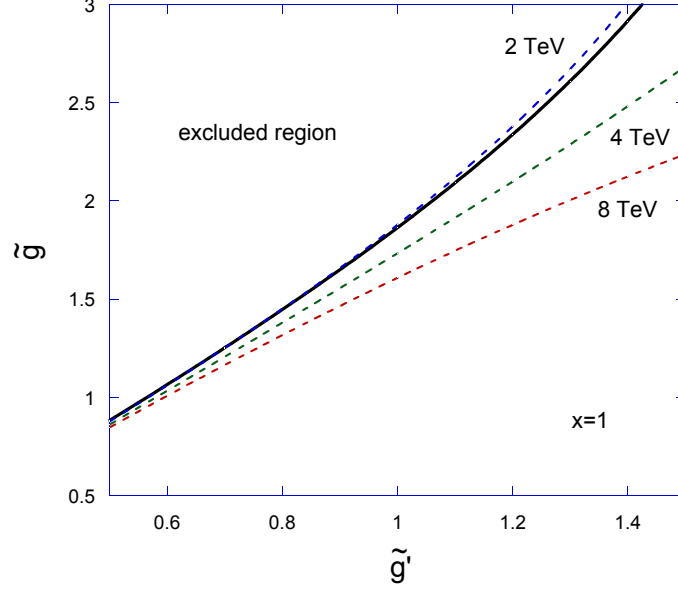


FIG. 6: Contours for m_1 , the mass of lightest exotic gauge boson in the G_2 model, assuming $x = 1$. For this choice, $m_1 = m_{1/2}$. The 95% C.L. exclusion region is also shown.

$SU(2)_W \times U(1)_Y$ subgroup. This corresponds precisely to our choice in Eqs. (2.1) and (2.2).

In this section, we will consider the case where the scale of the vector-like matter is higher than the unification scale, $M_F > M_U$. In this limit, the vector-like matter may be integrated out of the theory, which simplifies the discussion of the heavy gauge boson decays. We construct a low-energy effective Lagrangian following the same procedure described in Section III. For definiteness, let us assume $n_F = 1$. Starting with the Lagrangian in Eq. (2.4), one may read off the equation of motion for ψ (or $\bar{\psi}$). To lowest order in \not{D}/M_F , the equation of motion for ψ is solved by

$$\psi = -\frac{1}{M_F} \left(1 + \frac{i \not{D}}{M_F} \right) [\Sigma \lambda^\ell \ell_R + \chi \lambda^e e_R] . \quad (4.1)$$

Substituting this into Eq. (2.4) and discarding higher-order terms leads to the effective Lagrangian

$$\mathcal{L}_{eff} = \frac{1}{M_F^2} (\bar{e}_R \lambda^{e\dagger} \chi^\dagger) i \not{D} (\Sigma \lambda^\ell \ell_R) + \frac{1}{M_F^2} (\bar{e}_R \lambda^{e\dagger} \chi^\dagger) i \not{D} (\chi \lambda^e e_R) + \text{h.c.} . \quad (4.2)$$

If we had never mentioned vector-like matter, we could instead have postulated that new physics at a cut off scale M_F generates higher-dimension operators, including those that lead to exotic gauge boson decays. Symmetry and power-counting would have led us to Eq. (4.2)

directly, with perhaps a different label for the unknown couplings in flavor space. It is worth noting that both terms in Eq. (4.2) involve the field χ which is not present in the minimal SU(3) model; this leads to gauge boson decay via operators of much lower order than those mentioned in Ref. [3].

Setting either some or all of the Higgs fields in Eq. (4.2) to their vevs leads to operators that contribute to the decays of the new scalar or vector states in the model. We focus the present discussion on the exotic gauge bosons. While one could imagine similar theories with different symmetry breaking sectors, the gauge boson content of both our SU(3) and G_2 models are, by definition, model independent. The interesting portion of the gauge boson matrix in the SU(3) model can be represented as

$$A^a T^a \supset \begin{pmatrix} \mathbf{0} & X \\ X^\dagger & 0 \end{pmatrix}, \quad (4.3)$$

where $X \sim \mathbf{2}_{\mathbf{3}/2}$. Substituting into Eq. (4.2), one may extract the X -fermion-fermion vertex:

$$\mathcal{L}_X = g_3 x \left(\frac{M}{M_F} \right)^2 \bar{e}_R [\lambda^{e^\dagger} \not{X}^\dagger \lambda^\ell] \ell_R + \text{h.c.} \quad (4.4)$$

Notice that the coupling vanishes when $M_F \rightarrow \infty$ or $x \rightarrow 0$.

In the G_2 model, the $\mathbf{2}_{\pm\mathbf{3}/2}$ gauge bosons also decay as a consequence of the operators in Eq. (4.2); the implicit SU(3) indices running from 1 to 3 in this expression should simply be promoted to G_2 indices running from 1 to 7. In the case of the G_2 model, we will not assume the presence of vector-like matter, and pursue the simpler approach of studying possible operators that may arise at a cut off M_F . The operators in Eq. (4.2) do not allow the new $\mathbf{2}_{\pm\mathbf{1}/2}$ and $\mathbf{1}_{\pm\mathbf{1}}$ G_2 gauge bosons to decay to Standard Model and/or neutral particles. We find that the simplest way to remedy this is to introduce a new singlet fermion ν_R , with the possible effective interactions

$$\mathcal{L}_{eff} = \frac{1}{M_F^2} \bar{\nu}_R^c (\chi^T S) \not{D} (\Sigma \ell_L) + \frac{1}{M_F^2} \bar{\nu}_R^c \text{Tr} [\Sigma^T S \not{D} \Sigma \epsilon] e_R^c + \text{h.c.}, \quad (4.5)$$

where S and ϵ are the matrices that were used in constructing gauge invariant operators in Section II B. In this expression, we let $\ell_L = (\nu_L, e_L^-)$ represent the Standard Model lepton doublet with hypercharge $-1/2$. Working in the basis of G_2 generators given in the

Appendix, we identify

$$A_{\mathbf{2}_{1/2}} = \begin{bmatrix} (A^{13} + iA^{14})/\sqrt{2} \\ (A^{11} + iA^{12})/\sqrt{2} \end{bmatrix} \quad A_{\mathbf{1}_{-1}} = (A^9 + iA^{10})/\sqrt{2} . \quad (4.6)$$

Eq. (4.5) then yields the effective interactions

$$\mathcal{L}_{eff} = -\frac{g_2 x}{\sqrt{12}} \left(\frac{M}{M_F} \right)^2 \bar{\nu}_R^c \mathcal{A}_{\mathbf{2}_{1/2}}^i \epsilon_{ij} \ell_L^j - \frac{g_2 x}{\sqrt{3}} \left(\frac{M}{M_F} \right)^2 \bar{\nu}_R^c \mathcal{A}_{\mathbf{1}_{-1}} e_R^c + \text{h.c.} \quad (4.7)$$

Again the decays vanish as $M_F \rightarrow \infty$ or $x \rightarrow 0$. It is worth mentioning that the additional right-handed singlet in the G_2 model may provide a possible dark matter candidate, but it is clearly premature to pursue that issue in detail here.

As mentioned in the introduction, the exotic gauge boson states in the $SU(3)$ and G_2 model can be long-lived if M_F is sufficiently high. In the case of the $\mathbf{2}_{\pm 3/2}$ gauge bosons, which is particularly interesting since it contains a doubly-charged bilepton state, we find

$$c \tau_X = 0.007 \text{ cm} \cdot \left(\frac{3 \text{ TeV}}{M_X} \right)^5 \cdot \left(\frac{M_F}{10 \text{ TeV}} \right)^4 \cdot \left[\frac{g_3^2 (1+x^2)^2}{4x^2} \right] , \quad (4.8)$$

where we have assumed mixing with one Standard Model generation only and taken $\lambda^e = \lambda^\ell = \sqrt{2} m_e / v$ as a representative choice. In the three-generation case, the couplings λ^e and λ^ℓ may lead to flavor-violating decays, since we have no restriction on their values in the basis in which the Standard Model lepton mass matrices are diagonal. If the lifetime is long, one could imagine that the branching fraction to lepton flavor-violating decay modes could be substantial without conflicting with low-energy bounds on lepton flavor-violating processes. The detailed collider physics of this possibility seems worthy further investigation.

V. CONCLUSIONS

In this paper we have considered two extensions of the $SU(3)$ electroweak model [2]. First, we introduced new fields (the χ field in the Higgs sector and the heavy vector-like fermions ψ) that provide an origin for higher-dimension operators that contribute to exotic gauge boson decays. We also considered a model that embeds $SU(3)$ in next smallest possible group, G_2 . The G_2 model includes exotic gauge bosons that are lighter than those of the $SU(3)$ model by a factor of $\sqrt{2/3}$ and may be somewhat easier to produce at future collider experiments.

We analyzed the experimental constraints on these models using precision electroweak observables, following the approach of Csáki, *et al.* [3]. We noted that measurements of the W mass from LEP II leads to improved bounds on the original SU(3) electroweak model, but that the effect on the allowed parameter space is small. In our models, the dependence on a new parameter x , which encodes a ratio of Higgs field vevs, has a more noticeable effect on the region of parameter space that is excluded by electroweak constraints. However, the smallest possible values of the unification scale and the $\mathbf{2}_{3/2}$ gauge boson mass can only be slightly improved using this additional degree of freedom. The constraints on the G_2 model were qualitatively similar to those for the SU(3) model.

We have also discussed the higher-dimension operators that contribute to the decay of otherwise exotic stable states, focusing specifically on the gauge boson sector. In the SU(3) model, we showed how these operators are generated by explicitly integrating out a sector of vector-like fermions; in the G_2 model, we construct similar operators directly, and found that a viable model requires that we introduce a phenomenologically harmless, singlet fermion in the low-energy spectrum. In both models, otherwise stable gauge bosons can be arbitrarily long-lived, if the cut-off (or vector-like) scale M_F is sufficiently high.

A natural direction for future study is the detailed collider physics of the exotic gauge bosons in these models. A collider study of the production and detection of the extra gauge bosons in the G_2 model does not exist and is timely given that they may be within the reach of the LHC. The potential for long lifetimes and lepton-flavor violating decays may lead to unique signatures in TeV-scale collider experiments.

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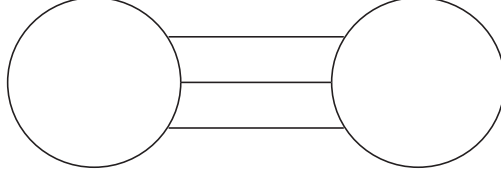


FIG. 7: Dynkin diagram for group G_2 . The two nodes indicate the two simple roots of the group. The triple line connecting them indicates that they are at an angle of $\theta = 5\pi/6$.

APPENDIX A: GENERATORS OF G_2

We use the Dynkin diagram as a starting point for analyzing the group G_2 . The Dynkin diagram encodes the simple root structure of the group, and provides all the information needed to construct its generators.

As indicated in Fig. 7, the group G_2 has two simple roots, at a relative angle of $\theta = 5\pi/6$. We may fix the simple roots as

$$\alpha^1 = (0, 1) \tag{A1}$$

$$\alpha^2 = \left(\frac{\sqrt{3}}{2}, -\frac{3}{2} \right). \tag{A2}$$

Applying the standard procedure (see Ref. [12]), we obtain the positive roots $\{\alpha^1, \alpha^2, \alpha^1 + \alpha^2, 2\alpha^1 + \alpha^2, 3\alpha^1 + \alpha^2, 3\alpha^1 + 2\alpha^2\}$. We also obtain the two fundamental weights

$$\mu^1 = \left(\frac{\sqrt{3}}{2}, \frac{1}{2} \right) \tag{A3}$$

$$\mu^2 = (\sqrt{3}, 0). \tag{A4}$$

The 7-dimensional, fundamental representation of G_2 corresponds to the fundamental weight μ^1 . The complete set of weights for this representation are given by $\{0, \pm\alpha^1, \pm(\alpha^1 + \alpha^2), \pm(2\alpha^1 + \alpha^2)\}$. We use this root-weight analysis to construct generators for the fundamental representation. In a convenient basis, the first eight generators are given by

$$T^a = \frac{1}{2\sqrt{2}} \begin{pmatrix} \lambda_a & 0 & 0 \\ 0 & -\lambda_a^* & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad a = 1 \dots 8, \tag{A5}$$

where λ_a represents the eight Gell-Mann matrices. The remaining six generators can be written

$$T^9 = \frac{1}{2\sqrt{6}} \begin{pmatrix} 0 & -i\lambda_2 & \sqrt{2}e_3 \\ i\lambda_2 & 0 & \sqrt{2}e_3 \\ \sqrt{2}e_3^T & \sqrt{2}e_3^T & 0 \end{pmatrix}, \quad (\text{A6})$$

$$T^{10} = \frac{1}{2\sqrt{6}} \begin{pmatrix} 0 & -\lambda_2 & i\sqrt{2}e_3 \\ -\lambda_2 & 0 & -i\sqrt{2}e_3 \\ -i\sqrt{2}e_3^T & i\sqrt{2}e_3^T & 0 \end{pmatrix}, \quad (\text{A7})$$

$$T^{11} = \frac{1}{2\sqrt{6}} \begin{pmatrix} 0 & i\lambda_5 & \sqrt{2}e_2 \\ -i\lambda_5 & 0 & \sqrt{2}e_2 \\ \sqrt{2}e_2^T & \sqrt{2}e_2^T & 0 \end{pmatrix}, \quad (\text{A8})$$

$$T^{12} = \frac{1}{2\sqrt{6}} \begin{pmatrix} 0 & \lambda_5 & i\sqrt{2}e_2 \\ \lambda_5 & 0 & -i\sqrt{2}e_2 \\ -i\sqrt{2}e_2^T & i\sqrt{2}e_2^T & 0 \end{pmatrix}, \quad (\text{A9})$$

$$T^{13} = \frac{1}{2\sqrt{6}} \begin{pmatrix} 0 & -i\lambda_7 & \sqrt{2}e_1 \\ i\lambda_7 & 0 & \sqrt{2}e_1 \\ \sqrt{2}e_1^T & \sqrt{2}e_1^T & 0 \end{pmatrix}, \quad (\text{A10})$$

$$T^{14} = \frac{1}{2\sqrt{6}} \begin{pmatrix} 0 & -\lambda_7 & i\sqrt{2}e_1 \\ -\lambda_7 & 0 & -i\sqrt{2}e_1 \\ -i\sqrt{2}e_1^T & i\sqrt{2}e_1^T & 0 \end{pmatrix}, \quad (\text{A11})$$

where e_i are the unit vectors

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \quad (\text{A12})$$

The SU(3) subgroup relevant to our earlier discussion is formed by the T^a for $a = 1 \dots 8$. In this basis, the SU(3) decomposition of the fundamental representation of G_2 is clear:

$$\mathbf{7} = \begin{pmatrix} \mathbf{3} \\ \bar{\mathbf{3}} \\ \mathbf{1} \end{pmatrix} \quad (\text{A13})$$

The generators T^a for $a = 9 \dots 14$ can be rewritten as

$$T^a = \frac{1}{2\sqrt{6}} \begin{pmatrix} 0 & M(\chi_a) & \sqrt{2}\chi_a \\ M(\chi_a)^\dagger & 0 & \sqrt{2}\chi_a^* \\ \sqrt{2}\chi_a^\dagger & \sqrt{2}\chi_a^T & 0 \end{pmatrix}, \quad (\text{A14})$$

where $M(\chi_a)$ is a three-by-three matrix defined by

$$M(\chi)^{ij} = -\epsilon^{ijk} \chi_k^*, \quad (\text{A15})$$

and where $\chi_9 = e_3$, $\chi_{10} = i e_3$, $\chi_{11} = e_2$, $\chi_{12} = i e_2$, $\chi_{13} = e_1$, and $\chi_{14} = i e_1$. This form indicates that the decomposition of the 14-dimensional, adjoint representation of G_2 under $SU(3)$ is

$$\mathbf{14} = \mathbf{8} + \mathbf{3} + \bar{\mathbf{3}}, \quad (\text{A16})$$

where the χ_a represent a basis for the 3-dimensional representation, and the $\bar{\mathbf{3}}$ is related to the $\mathbf{3}$ by complex conjugation.

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